MATH 320 Unit 3 Exam Name:

Instructions:

Please write your answers on separate paper. Please write clearly and legibly, using a large font and plenty of white space (I need room to put my comments). Staple all your pages together, with your problems in order, when you turn in your exam. Please do not write under the staple. Make clear what work goes with which problem. Put your name or initials on every page. To get credit, you must show adequate work to justify your answers. If unsure, show the work. No outside materials are permitted on this exam – no notes, papers, books, calculators, phones, smartwatches, or computers – only pens and pencils, and your coursepack. You may use any result in the coursepack (whether boxed or an exercise). However, you must cite it, and you may not use it to trivialize an exam question (e.g. to prove itself or a portion of itself or a special case of itself). Each problem is out of 10 points, 100 points maximum. You have 75 minutes.

1. Let R be an integral domain, and let $f(x) \in R[x]$. Prove that $f(x)$ is a unit in $R[x]$, if and only if, $f(x) = a_0$, where a_0 is some unit of R.
2. Let $f(x), g(x) \in \mathbb{Q}[x]$, where $f(x) \neq 0$. Prove that $\gcd(f(x), g(x)) = \gcd(f(x), g(x) + xf(x))$.
3. Working in $\mathbb{Q}[x]$, set $f(x) = x^3 - 2x^2 + x - 2$ and $g(x) = 2x^4 + x^3 + 3x^2 + x + 1$. Use the $\mathbb{F}[x]$ Euclidean algorithm to find $\gcd(f(x), g(x))$ and also to find $u(x), v(x) \in \mathbb{Q}[x]$ satisfying $u(x)f(x) + v(x)g(x) = \gcd(f(x), g(x))$.
4. Set $R = \mathbb{Z}_{49}$. Find $f(x) \in R[x]$ where $\deg(f(x)) = 2$ and also $f(x)$ is a unit.
5. Find all monic irreducible polynomials of degree 3, in $\mathbb{Z}_2[x]$. Be sure to justify your answer.
6. Find a polynomial of degree 5 in $\mathbb{Z}_3[x]$ that induces the zero function on $\mathbb{Z}_3$.
7. Let R be a commutative ring with identity. Let $a \in R$ satisfy $a^4 = 0_R$. Prove that $1_R + ax$ is a unit in $R[x]$.
8. Set $R = \mathbb{Z}_7[x]$, $f(x) = x^3 + [3]x^2 + [2]x + [6]$, and $g(x) = x^3 - [1]$. Use the Euclidean algorithm to find $\gcd(f(x), g(x))$.
9. Prove the Degree Sum Theorem.
10. Prove the Remainder Theorem.